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Review Paper

Mathematical Modelling of the Ultrafiltration Process for Radioactive Waste: A Review

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Highlights

- A comprehensive review of mathematical models for ultrafiltration in radioactive waste treatment, highlighting pathways to enhance efficiency and safety.
- Machine and deep learning models are identified as powerful tools for optimizing process parameters and improving predictive accuracy.
- Integration of dynamic modeling and real-time optimization offers promising solutions to critical challenges like membrane fouling and system scalability.
- The analysis covers models specifically designed for actinide separation and trace metal removal from complex waste streams.

Graphical abstract

Abstract

The management of radioactive waste generated during nuclear energy production remains a serious problem requiring advanced treatment methods such as ultrafiltration. This review examines the latest advances in mathematical modeling specifically related to the ultrafiltration process used for processing radioactive waste. The review summarizes key mathematical models (bulk flow, concentration polarization, gel polarization, osmotic pressure, and resistance), as well as their applications, limitations, and forecasting capabilities. The results show that although traditional models provide valuable information about the dynamics of ultrafiltration, new approaches, including machine learning and dynamic modeling, have significant potential to improve the efficiency of the process and the accuracy of forecasting. However, the problems of membrane fouling, scaling, and long-term performance under different working conditions have not been effectively solved. The significance of incorporating advanced computational methods in conjunction with experimental validation is underscored in the present review in order to increase the performance of the ultrafiltration process in radioactive waste treatment. The findings present a potential approach for enhancing resistance of nuclear waste management systems against radiation degradation, complementing the other strategies to increase sustainability, safety, and efficiency of the nuclear fuel cycle.

1. Introduction

The handling of radioactive waste can be one of the most difficult tasks that must be performed in order to create sustainable energy from nuclear. Although nuclear power is a low-carbon energy, it produces intensely radioactive waste that must be managed, treated, and disposed of safely with minimum impact on the environment and health risks to humans.

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Over the years, ultrafiltration has become the preferred method of recycling liquid radioactive waste, notable for its remarkable efficiency in removing suspended particles and pollutants from the waste stream.

While traditional dynamic (mechanistic) models have long been employed in ultrafiltration (UF) for the treatment of radioactive waste, they are often constrained by oversimplifications and assumptions and the inability to adapt to nonlinear and transient system behaviors.

Conversely, models constructed based on machine learning (ML) and hybrid models have become effective tools for data-driven predictions, monitoring, and optimization.

These models can capture nonlinearities in operating parameters even with limited or noisy data.

Due to the lack of a comparative review focused on how ML-based and classical models perform in UF processing of radioactive waste, this review aims to address this gap by:

1. Comparing mechanistic and ML-based approaches in terms of computational feasibility and operational relevance
2. Highlight hybrid modeling trends as a bridge between interpretability and performance.

1.1. Literature Search Strategy

A systematic search was conducted using the Scopus database on February 1, 2025, with the following Boolean query:

TITLE-ABS-KEY (*mathematic* AND *model* AND *ultrafiltration*) AND (LIMIT-TO (SRCTYPE, "j")) AND
(EXCLUDE (SUBJAREA, "BIOC") OR EXCLUDE (SUBJAREA, "AGRI") OR EXCLUDE (SUBJAREA, "PHAR")
OR EXCLUDE (SUBJAREA, "IMMU") OR EXCLUDE (SUBJAREA, "BUSI") OR EXCLUDE (SUBJAREA, "SOCI")
OR EXCLUDE (SUBJAREA, "ARTS") OR EXCLUDE (SUBJAREA, "ECON") OR EXCLUDE (SUBJAREA, "DENT")
OR EXCLUDE (SUBJAREA, "PSYC") OR EXCLUDE (SUBJAREA, "VETE"))
AND (LIMIT-TO (LANGUAGE, "English")) AND (LIMIT-TO (DOCTYPE, "ar")).

The search returned 528 records, which were screened in accordance with PRISMA guidelines. The PRISMA flow diagram is shown in Fig. 1. After removing duplicates, conference abstracts, and articles irrelevant to radioactive waste treatment or mathematical modeling, a final set of 16 articles was selected for in-depth review. The inclusion criteria included:

[1] Use of mathematical or computational modeling in UF systems
[2] Explicit application to radioactive or nuclear wastewater
[3] Availability of full text in English peer-reviewed journals

Exclusion criteria included:

[1] Purely experimental studies without modeling
[2] Reviews not focused on UF or radioactive waste
[3] Non-peer-reviewed literature

1.2. Evolution and Trends in Ultrafiltration Modeling for Radioactive Waste Treatment

1.2.1. Growing Research Interest

The number of publications on ultrafiltration (UF) modeling for radioactive waste has surged over the past decade. This trend is driven by increased nuclear energy production, which has amplified radioactive waste volumes, necessitating advanced treatment methods.

In addition, stricter environmental standards demand efficient radionuclide removal technologies due to the potential hazards it poses to health and the environment in general. [8].

1.3. Classification of Modelling Methods in Ultrafiltration for Radioactive Waste Treatment

The modelling of ultrafiltration processes in the context of radioactive waste can be broadly categorized into classical equation-based approaches and modern data-driven techniques. Increasingly, hybrid models are also emerging, combining the strength of both technologies.

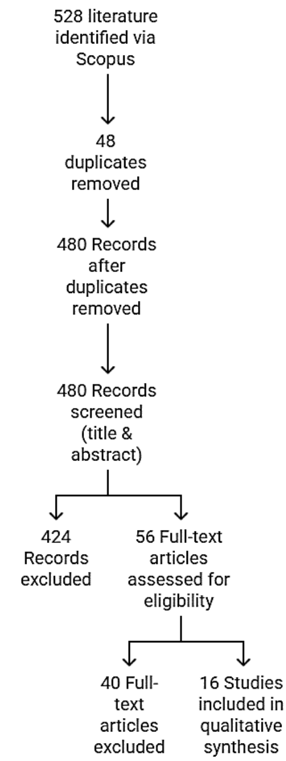

Fig. 1. PRISMA flow diagram.

1.4. Classical Models of the Ultrafiltration Process

Classical models provide fundamental insights into ultrafiltration dynamics. Such models are required to predict permeate flux, solute concentration on the membrane surface, and membrane performance under different operational conditions.

These models incorporate:

1. Darcy's law for transmembrane flux relationships.
2. Film theory for concentration polarization.
3. Resistance in series models to characterize different fouling mechanisms.
4. Osmotic Pressure model.
5. Mass transfer and fluid dynamic equations to describe solute transport and hydrodynamic flow.

A quick chronological excursion showed that in 1885, Darcy developed a mathematical model to describe volumetric flow in applications that involve normal flow filtration (i.e., dead-end filtration, DEF); continuous rotary filtration using auxiliary filters or sterile filtration; sediment filtration when Reynolds numbers are less than 1 (i.e., laminar flow) and inertial effects are negligible; and local membrane flow in cross-flow (i.e., tangential flow, TFF) modes.

Darcy's law states that the instantaneous rate of filtration, *dv{t}/dt*, and the flux of permeate, *J* (dm^3/m^2-h), of viscosity μ through a medium of permeability *k* and cross-sectional area *AM* are proportional to a differential pressure drop *dP* through thickness *dz*.

$$J = \frac{1}{A_M}\frac{dV\{t\}}{dt} = -\frac{kdP}{\mu dz} \tag{1}$$

The resistance of a membrane varies with its permeability L_p which is given by

$$L_P = \frac{J}{\Delta P} \tag{2}$$

However, the mathematical model that describes the relationship between the permeate flux and the feed pressure (i.e., mean transmembrane pressure) was not known. This was later discovered by Kozeny in 1927 and further modified by Carman in 1938.

$$J = L_P \Delta P = \frac{\Delta P}{\mu R_m} \tag{3}$$

where;

J is the flux (volumetric rate per unit area).
ΔP is the transmembrane pressure,
Lp is the permeability coefficient of the membrane.
μ is the solvent viscosity, and
Rm is the hydrodynamic resistance of the membrane.

1.4.1. Concentration Polarization (CP)

CP happens on both faces of the membrane due to the permeation of the solvent through the membrane and retention of the solute, resulting in solute accumulation and local concentration buildup.

CP is a reversible phenomenon that leads to a local increase in solute concentration near the membrane surface, which leads to a decrease in flow due to increased hydrodynamic resistance and osmotic pressure.

Fouling, however, is the irreversible diminution of flux due to particles or organic particles depositing on the membrane surface.

1.4.2. Gel Polarization model

Gel polarization occurs when, under high-pressure operation, the concentration at the membrane surface rises to the point of gel precipitation. When the membrane surface concentration becomes extremely high and a gel layer forms, any further increase in pressure no longer results in an increase in flux. At this point, cg is used instead of cm (concentration at the membrane surface).

$$J_{v,\lim} = k \ln \frac{C_g}{C_b} \tag{4}$$

$J_{v,\lim}$ = limiting flux
C_g = Gel layer concentration

As bulk concentration approaches the gel layer concentration, the limiting flux approaches zero.

The model is based on the following assumptions:

1. At a steady state, the quantity of solutes conveyed by the solvent to the membrane is equal to those that diffuse back.
2. The solute concentration in the permeate can be ignored because of high rejection of ultrafiltration for macromolecules.
3. The concentration of the gel layer is constant and dependent only on the type of solutes and membrane used.

1.4.3. Osmotic Pressure Model

The osmotic pressure model was derived in 1958 by Kedem and Kachalsky. The model describes the permeate flux with the osmotic pressure difference between the two sides of the membrane. Osmotic pressure exists at concentration polarization and hence high surface concentration.

$$J = \frac{|\Delta P| - |\Delta \pi|}{\mu(R_m + R_s)} \tag{5}$$

$\Delta\pi$= Difference in osmotic pressure across the membrane
R_s=Reversible (concentrated layer, filter cake) and sometimes irreversible (foulants) deposition of solute (or solids) onto the membrane surface.

1.4.4. Resistance Models

When the osmotic pressure term in the equation (7) is neglected, the result is the resistance model given by:

$$J = \frac{|\Delta P|}{\mu(R_m + R_s)} \tag{6}$$

$$R_s = \alpha \frac{m_p}{A_m} \tag{7}$$

M_p is the mass of deposited particles, A_m is the membrane area, and α is the specific resistance of the deposit.

$$J = \frac{|\Delta P|}{\mu(R_m + R_{bl} + R_g)} \tag{8}$$

1.5. Considerations for Radionuclide Modeling in UF systems

Here, issues surrounding radionuclide-specific properties, such as ionic strength, radiolysis products, high valence states, and radiation-induced degradation of the membrane, are mapped into fundamental assumptions in classical UF models. These include:

[1] High ionic strength, typical of radioactive elements, impacts the osmotic pressure gradients as well as floc structure, thus affecting the compressibility and hydraulic resistance of the gel layer.

[2] In case of exposure to alpha or beta emitters that are able to degrade polymeric membranes, the assumption in the resistance-in-series model (i.e., a fixed membrane resistance Rm) may not be valid.

Table 1 below summarizes in what way major classical UF models must be modified when they are applied to radioactive waste.

2. Analysis of Classical Mathematical Models in Ultrafiltration

2.1 Overview of Classical Modeling Approaches

Classical mathematical models can be broadly divided into the mechanistic model and the empirical model.

Mechanistic models are composed of mass, momentum, energy, and transport laws.

They attempt to describe the physical behavior of ultrafiltration processes using differential equations (DEs) and algebraic equations (AEs). For example:

- Mass transport (e.g., convection-diffusion equations)
- Fluid dynamics (e.g., Navier-Stokes for flow regimes)
- Sorption kinetics (e.g., Langmuir isotherms)
- Membrane resistance (e.g., Darcy's law, resistance-in-series models)

Empirical models are based on observed data rather than physical laws. They fit equations (often polynomial, exponential, or power-law) to experimental results and are often used when the physical mechanism is complex or unknown. Table 2 below shows the summary of classical UF models.

Table 1
Applicable modifications to Classical UF models for radioactive waste.

Model Name	Assumptions	Impact of radionuclide
Gel layer	Stable gel structure Uniform compressibility	radiolytic decomposition changes the compactness of gel matrices
Osmotic Pressure model	Constant Osmotic gradient No ion-exchange	The increase in ionic strength due to radionuclides alters the osmotic pressure. Increasing the ionic strength of the bulk solution decreases osmotic pressure [3].
Resistance-in-series	Fixed R_m No membrane aging	Radiation damages polymetric membranes, thus making R_m time variant.[31].
Concentration Model	Solute rejection is only size dependent	Radiolysis changes solute speciation [17]. Radiolysis also causes ion pairing which can alter diffusion coefficient D, C_m and C_p. $J = k_s \ln\left(\frac{c_m - c_p}{c_b - c_p}\right)$ where $k_s = D/\delta$ Cm = concentration at the membrane surface. C_p = solute concentration

Table 2
Summary of Key Differential and Algebraic Equations in Classical Mechanistic UF Models.

Model Type	Governing Equations	Purpose
Bulk Flow	Darcy's law: $J = L_p \Delta P = \dfrac{\Delta P}{\mu R_m}$	Predicts permeate flux (J) under transmembrane pressure (ΔP).
Concentration polarization	Film Theory: $J = k_s \ln\left(\dfrac{c_m - c_p}{c_b - c_p}\right)$	Estimates solute accumulation at membrane surface (Cm).
Gel Polarization	Modified flux equation $J_{v,\lim} = k \ln \dfrac{C_g}{C_b}$	Describes flux limitation due to gel layer formation.
Osmotic Pressure	Kedem Katchalsky: $J = L_P(\Delta P - \sigma \Delta \pi)$	Accounts for osmotic pressure effects.
Resistance Model	Modified from osmotic pressure model: $J = \dfrac{\lvert \Delta P \rvert}{\mu(R_m + R_{bl} + R_g)}$	Accounts for membrane resistance due to the mass of deposited particle
Fouling Kinetics Model	Cake growth over time $\dfrac{dR_c}{dt} = \alpha J C_b$ R_c = cake resistance α = specific resistance C_b = bulk solute concentration	Accounts for the formation of cake layer on membrane over time (fouling).

2.2. Finite Difference Method

Finite Difference Method (FDM), a classical, deterministic numerical technique used to approximate solutions to differential equations by discretizing them over a structured grid, was used by [32] to solve the governing transport and momentum equations, which revealed that pulsed flow significantly enhances flux with only a modest increase in energy consumption. FDM-based models overcame key limitations of classical differential-algebraic equation (DAE) approaches by enabling high-resolution simulations of transient UF phenomena through discretized PDE solutions**.**

2.3. Finite Element Method

FEM-based models offer a compelling alternative to classical DAE-based approaches by enabling high-fidelity simulation of complex ultrafiltration (UF) systems. Their advantage is in the ability to capture the regional and temporal phenomena such as the saline spacers wake, concentration polarization, and fouling layer development, which are usually oversimplified in lumped-parameter models.

For example, [22] used FEM to simulate tubular UF of bentonite suspensions, accurately modeling flux decline and fouling layer growth under varying operational conditions. Similarly, [27] developed a two-dimensional FEM-based model to resolve coupled flow and solute transport, enabling precise predictions of permeate behavior and solute accumulation in membrane channels. Although these simulations provided flux prediction errors within 5%, they required computational runtimes exceeding 72 hours, highlighting the trade-off between accuracy and computational feasibility. As such, FEM remains a powerful tool for research and module design, but its computational burden currently limits its adoption in real-time control or industrial-scale applications.

2.4. Computational Fluid Dynamics CFD

CFD is a powerful numerical modeling technique that solves the Navier-Stokes, continuity, and mass transport (convection–diffusion) equations over discretized spatial domains. In this way, CFD provides a tool for very accurate modeling of fluid flow, concentration polarization, and fouling in ultrafiltration modules. CFD uses the FDM method to solve fluid flow problems.

While CFD provides unparalleled insights into localized phenomena (e.g., spacer-induced vortices), its computational expense motivates hybrid approaches, e.g., surrogate ANNs trained on CFD data to enable real-time control in industrial UF systems [16].

Table 3
List of Symbols and Units.

Symbols	Definition	Units
J	Permeate flux	$L.m^{-2}.h^{-1}$
ΔP	Transmembrane pressure difference	$Pa(N.m^{-2})$
R_m	Membrane resistance	m^{-1}
R_f	Fouling resistance	m^{-1}
R_t	Total resistance	m^{-1}
μ	Dynamic viscosity	$Pa.s$
C_b	Bulk solute concentration	mg/L
C_w	Solute concentration at membrane surface	mg/L
D	Diffusion coefficient	$m^{-2}.s^{-1}$
k	Mass transfer coefficient	$m.s^{-1}$
δ	Boundary layer thickness	m
A	Membrane surface area	m^2

3. Modern Modeling Methods used in Ultrafiltration of Radioactive Waste

Over the years the number of publications using modern modeling methods in modeling ultrafiltration of radioactive waste has grown. This is due to the development of the nuclear industry, the growth of the amount of waste, the need for process optimization, and, as a consequence, the increase in the relevance of this topic.

Modern modeling methods used in ultrafiltration of radioactive waste include:
1. Machine Learning (ML) and Neural Network (NN)-based Models
2. Statistical/Empirical Methods (e.g., RSM, regression)
3. Hybrid Modeling Approaches

3.1. Machine Learning (ML) and Neural Network (NN)-based Models

Neural Networks (NN) and Machine Learning (ML) have become powerful tools for modeling UF systems without requiring detailed physical equations. These models learn patterns in the data through supervised training, making them well-suited to handle complex, nonlinear behaviors typical of radioactive waste treatment.

Other modern modeling types that are ML and NN based are shown below in Table 4.

Table 4
ML and NN based models in UF processing.

Method	Description	Use Case in UF	Reference
Support Vector Machines (SVM)	Supervised ML model for classification/regression using hyperplanes	Predicting flux decline, fouling categorization, separation efficiency	[34] suggest SVM as future improvement to RSM in MEUF for uranium removal
Genetic Algorithms (GA) / Evolutionary Algorithms	Nature-inspired optimization for multi-objective problems	Optimizing membrane design, operating conditions (TMP, pH)	[21]
Fuzzy Logic Models	Rule-based systems handling uncertainty and linguistic inputs	Modeling rejection and fouling under uncertain feed conditions	[28]., used fuzzy logic in micellar enhanced UF
Deep Learning (DL)	Multi-layered neural networks (e.g., CNNs, LSTMs)	Advanced applications: SEM image analysis, real-time dynamic prediction	[18] *IEEE Access*: deep learning for uremic toxin removal modeling
Hybrid Models (Mechanistic + ML)	Combining first-principles with ML correction or learning residuals	Correcting mechanistic predictions of flux/fouling in real-time	[29] suggest extending their mechanistic complexation UF model with dynamic/ML tools. [1] optimized crossflow UF using hybrid network.
Digital Twins	Real-time simulation models replicating physical systems using live data	Smart monitoring of UF in nuclear waste facilities	[23] applied digital twins in process control

3.1.1. Performance Metrics for ML and DF models

Evaluation of machine learning-based models, including both ML and DL models applied in UF modeling of radioactive waste, is accomplished with the use of standard regression-based metrics: coefficient of determination (R^2), root mean square error (RMSE), mean absolute error (MAE), and mean bias error (MBE). For example, in the study by [6], a model was presented for predicting permeate flux in vacuum membrane distillation (VMD) for radioactive wastewater treatment using ANN. The ANN model demonstrated high accuracy, achieving an R^2 value of 0.975 and a maximum RMSE of 1.83%, indicating strong predictive performance.

While direct applications of machine learning (ML) in radioactive ultrafiltration (UF) are few, various basic modeling frameworks provide for useful perception. [5] did evaluate the performance of a filtration model. In a radioactive waste context, they compared predicted dimensionless pressure loss values to experimental ones. The model achieved RMSE values of 0.11 along with 0.12 for the deterministic and stochastic models, respectively, which indicates that it can strongly predict as well as capture pressure loss dynamics when operating conditions vary.

Model training typically involves dataset partitioning into training (70-80%), validation (10-15%), and testing (10-20%) subsets. K-fold cross-validation, especially with k=5 or 10, is frequently employed to minimize overfitting. For example, in the study by [1] ML based model was employed to optimize the vitrification process of nuclear waste. They used five-fold cross validation, with an 80-20 split on the dataset for each cross validation.

Input features of UF models such as transmembrane pressure, membrane area, flow velocity, pH, and solute characteristics are often normalized to improve convergence. This was demonstrated in the model by [19] which normalizes data via 0-1 scaling. It splits the dataset into 60% training and 40% testing sets. Threefold cross-validation tuned hyperparameters internally prior to testing. Output features vary depending on the modeling aim. It could be permeate flux, retention rate, or membrane resistance.

These strategies are now a standard practice in UF modeling using ML, and they do align so well with best practices for predictive modeling in related membrane separation applications.

3.2. Statistical/Empirical Methods

Statistical and empirical modeling techniques such as response surface methodology (RSM) and linear and multiple regression play a vital role in optimizing ultrafiltration (UF) processes, especially in cases where the mechanistic understanding is incomplete or when rapid experimental optimization is required.

These methods are especially effective when:

1. Designing experiments efficiently (DOE): These RSM approaches, such as central composite design (CCD), allow for structured experimentation and reduce the number of trials required. [34].
2. Key process variables: through statistical modeling we are able to identify the factors that are most significantly associated with system performance. For example, the work by [20] used RSM to recognize the importance of pH, pressure, and polymer/metal ratio in Sr/Cs removal.
3. Optimal conditions for separation without the need for complex transport equations: With this approach, detailed physical modeling can be avoided without sacrificing accuracy of the final results. [34] achieved 99% uranium rejection by statistical modeling and optimizing SDS concentration, pH, and pressure.

In radioactive waste treatment, experimentation may be expensive and hazardous, thus making this method highly adopted and highly practical.

3.3. Hybrid Modeling Approaches

To balance the interpretability of classical, mechanistic models with the flexibility of machine learning, researchers are increasingly exploring hybrid models, especially combining ANNs with evolutionary algorithms like genetic algorithms (GAs) for optimization. [24].

[1] combined ANN and GA to model and optimize UF performance in treating oily wastewater. Their model attained a decently high accuracy with a high R-value (i.e., coefficient of correlation) exceeding 0.99.

In ultrafiltration (UF) of radioactive waste, systems are controlled by complex and nonlinear phenomena that are often too complicated to model solely using classical transport equations [14][29].

Thus, hybrid models offer a practical and powerful compromise, particularly beneficial in scenarios with limited or hazardous experimentation [36] [7].

ANN and finite element method (FEM) models have both been successfully tested for the modeling of ultrafiltration (UF) in the literature; however, these models are used for different purposes. ANNs are excellent in optimizing operations such as permeate flow prediction, potential contamination, and process parameter optimization [6], especially when rapid decision-making is required. On the contrary, FEM remains indispensable for the design of fundamental modules and systems, offering a detailed understanding of localized flow dynamics, fouling layer evolution, and membrane-strut interactions [22] [27].

Recent research highlights the growing synergy between data-driven and mechanistic approaches. For example, [4] developed a hybrid mechanistic-ANN model that combines high-precision modeling with adaptive learning optimization and control of UF processes in water treatment, demonstrating the potential for real-time bridging and control in membrane systems. Such hybrid systems use FEM's physical precision and ANN's pattern recognition capabilities to solve multiscale and nonlinear problems in the treatment of radioactive waste.

The application of models that are solely ANN poses serious concerns, especially in nuclear waste recycling, where safety is paramount. These models, despite their effectiveness, often operate as "black boxes" and may lack physical consistency or violate environmental laws, a problem that has attracted the close attention of regulators in high-stakes sectors [2] [33].

For this reason, there is an increasing preference for physically limited or hybrid models to increase the reliability and interpretability of complex separation systems.

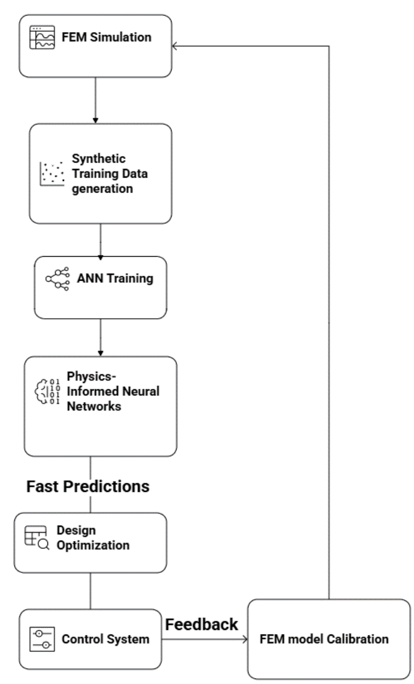

Fig. 2. Practical workflow for Hybrid ANN and FEM For Radioactive waste UF.

4. Recent models of Ultrafiltration process for radioactive wastes

Recent advances in ultrafiltration modeling for radioactive waste treatment have shifted from classical mechanistic models to modern approaches such as ANNs, RSM, and hybrid techniques. These methods offer improved accuracy, adaptability, and suitability for complex, nonlinear systems, especially where experimental data are limited or hazardous to obtain.

Radioactive Solutions Treatment by Hybrid Complexation-UF/NF Process

The study by [35] explores a hybrid ultrafiltration (UF) and complexation model for the removal of radioactive ions from liquid waste. This approach enhances separation efficiency by introducing macromolecular ligands that bind with radioactive ions, thereby improving membrane retention. The model demonstrated high decontamination factors and adaptability across low- and medium-level radioactive waste streams.

The hybrid UF-complexation model presented can be optimized for practical applications by performing computational simulations to improve membrane choice, ligand efficiency, and pH management and experimental validation under varied operational circumstances to find the best process configurations. Improvements in membrane material can also help reduce fouling and improve long-term stability.

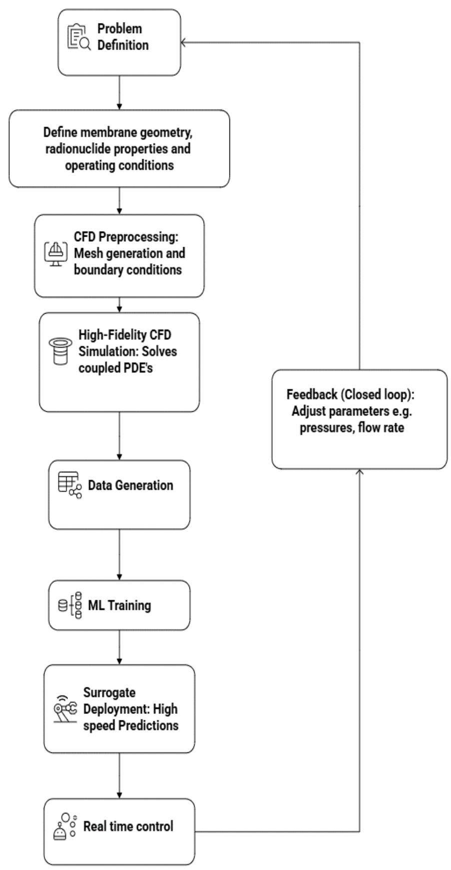

Fig. 3. Practical workflow for Hybrid ANN and CFD For Radioactive waste UF.

Maximizing Production Capacity from an Ultrafiltration Process at the Hanford Department of Energy Waste Treatment Facility

[10] conducted a study aimed at maximizing production capacity in an ultrafiltration process at the Hanford Department of Energy Waste Treatment Facility. The study employed a model based on a form of the film model to describe the flux rate over time and determine the optimal endpoint solids concentration (Cs). The model assumed constant concentration at the membrane wall and a fixed mass transfer coefficient, simplifying the analysis. However, these assumptions introduced errors due to the long time required for a steady-state cake layer to form and the changing fluid properties of the slurry throughout operation. Additionally, the model did not account for membrane fouling, a significant factor affecting ultrafiltration performance.

The research presented seems to establish a relationship between the concentration of the solid in the slurry and the flux. However, a few points were noted as areas that need investigation due to information gaps. The effects of slurry properties on the optimal Cs, the impact of membrane fouling, and the relationship between membrane surface area and endpoint solids concentration require further investigation. Future research should focus on developing more accurate models that incorporate long-term slurry property changes and cake layer formation, enhancing the reliability of ultrafiltration predictions. Moreover, extending the models to account for membrane fouling effects will improve their applicability to real-world ultrafiltration systems.

Optimization of Ultrafiltration Feed Conditions for HLW Filtration

Membrane-based separation processes play a crucial role in high-level waste (HLW) treatment, particularly at the Hanford Waste Treatment Plant (WTP). In this context, [13] evaluated two classical filtration models (the concentration polarization model and the resistance in series model) to optimize ultrafiltration feed conditions. These models provide a quantitative framework for determining the optimal sodium molarity and solids content to maximize throughput efficiency.

A significant strength of the study is that the resistance in-series model accurately represents real-world filtration behavior, incorporating factors such as membrane resistance and gel layer formation. This allows for better predictions of operational performance and throughput rates. Furthermore, the study challenges previous assumptions by demonstrating that higher throughput rates can be achieved through feed dilution rather than increasing sodium molarity.

However, the study does have limitations, as it assumes steady-state filtration conditions, which do not fully capture dynamic changes in sludge properties over extended filtration cycles. Furthermore, the model makes no consideration of membrane fouling, which is an important factor for long-term filtration operation.

The models presented in the study provide a valuable tool for process optimization by identifying optimal sodium molarity levels that maximize permeate and retentate throughput. Additionally, they offer insights into feed dilution strategies, reducing the solid concentration to improve filter performance. By predicting cycle times and filtration rates, these models aid in designing more efficient ultrafiltration systems for HLW treatment.

These models can be directly applied to real-world HLW treatment scenarios by reducing processing costs through optimized sodium concentrations, eliminating unnecessary evaporation steps, and enhancing operational efficiency by determining ideal transmembrane pressures and slurry compositions. Furthermore, they contribute to improving waste separation efficiency, ensuring that radionuclides are effectively removed before vitrification for long-term storage.

In order to improve the applicability of the study's models, future research needs to develop dynamic models that can integrate actual sludge property discrepancies and membrane fouling effects.

Furthermore, exploring hybrid modeling approaches that integrate machine learning with classical filtration models could further refine predictive optimization and adaptability to different waste streams.

Ultrafilter Conditions for High-Level Waste Sludge Processing

Ultrafiltration plays a crucial role in processing high-level waste (HLW) sludges, and the study by [14] provides a comprehensive and well-structured model to predict the flux of HLW sludges under varying operating conditions. Rheological properties, filter cake resistance, and gel layer effects are included in the model, and it can thus be employed as a useful tool in the optimization of ultrafiltration processes.

However, the model has some limitations. It assumes a fixed gel concentration at the membrane, which may not always be accurate. Membrane fouling phenomena are also ignored, the effect of which can be severe on long-term filtration performance. Finally, the model is developed using a small set of experimental data, which may not capture all possible operating conditions.

Despite the study being robust, it highlights several knowledge gaps that need further examination. These include:

1. The lack of experimental data at lower solids concentrations limits the model's accuracy in predicting performance under diluted conditions.
2. A limited understanding of membrane fouling mechanisms affects long-term filtration efficiency.
3. A need for a more detailed characterization of HLW slurry rheology, particularly at high solids concentrations where viscosity behavior becomes increasingly complex.

To further improve the model and its applicability, future research should expand experimental datasets to include lower solids concentrations for improved model accuracy. In addition, incorporating fouling effects into the model will help to better predict long-term ultrafiltration performance.

Combined Radioactive Liquid Waste Treatment Processes Involving Inorganic Sorbents and Micro/Ultrafiltration

[7] investigated the sorption behavior of 137Cs on natural zeolites, using the Langmuir and Freundlich models to model the sorption process. Their findings demonstrated a strong relationship between experimental and theoretical data, supporting the applicability of these models. However, each model has limitations: the Langmuir model assumes monolayer adsorption with constant adsorption energy, which may not accurately represent the complex sorption mechanisms in this system, while the Freundlich model, though better suited for non-ideal adsorption, is less precise in predicting sorption capacity at low concentrations.

The research offered insight on the application of natural zeolites for 137Cs removal as well as contribution to ultrafiltration process optimization. Use of these models to predict performance for pretreatment alternatives enables the optimization of pretreatment options to minimize waste generation and improve radionuclide removal. However, there are still information gaps that include the impacts of temperature, pH, and competing contaminants on sorption behavior.

Future research should focus on refining these models to incorporate additional factors influencing sorption, extending their applicability to real-world scenarios, and exploring alternative natural sorbents for radionuclide removal. By integrating more comprehensive sorption data, researchers can develop more robust models that improve the efficiency of ultrafiltration systems for radioactive waste treatment.

Sizing an Ultrafiltration Process that Will Treat Radioactive Waste

The model in this study by [9] is based on the assumption that the filtration processes can be modeled as a linear relationship between time and volume of permeate collected. This model, known as Geeting's curve, has been shown to have a high correlation with data from the Ultrafiltration Process (UFP) at Hanford. However, it is important to note that this model is only valid for a specific range of conditions and may not apply to all ultrafiltration processes. Additionally, the model assumes constant cake properties and does not account for changes in cake structure or composition over time.

The authors highlighted a key gap in understanding the mechanisms governing ultrafiltration systems at large scales. While multiple studies have been conducted on the proposed UFP at Hanford, discrepancies often arise due to issues of scale. Addressing these scale-up challenges is essential for developing models that can accurately predict ultrafiltration performance in industrial applications.

The models developed in this study will greatly enrich the ability to optimize the ultrafiltration process and can be extended to treat radioactive waste, industrial wastewater treatment, and the biopharmaceutical industry. By anticipating membrane performance and determining optimal operating parameters, these models can improve the process efficiency, cost-effectiveness, and waste reduction. However, more studies are needed to develop these models by considering the additional variables of membrane fouling, scaling, temperature, pH, and additives in order to apply them to other more general ranges of operation.

Mathematical model for the removal of trace metal by complexation-ultrafiltration

The study by [30] provides a comprehensive analysis of ultrafiltration modeling, incorporating physical parameters such as pore size distribution, molecular weight distribution of the ligand, and solute permeability. This approach enhances the understanding of filtration mechanisms compared to purely data-driven models. The modified model presented in the study demonstrates good agreement with experimental results, making it valuable for performance prediction.

One of the key findings is the model's limitation in accurately predicting rejection behavior when metal species exceed the ligand's complexing capacity. This is because the model primarily focuses on physical sieving mechanisms and does not fully account for complex physicochemical interactions. The study also relies on several assumptions, which include:

i. Instantaneous equilibrium, which may not always hold in real systems.

ii. The assumption that pressure drops due to fouling or concentration polarization are negligible, which may not be generalizable.

iii. The exclusion of hydroxide formation, despite the potential impact of pH fluctuations during the process.

Despite these strong points, the study detects several areas needing further research. First, incorporating dynamic models that account for time-dependent factors such as complexation kinetics, fouling, and concentration polarization would improve predictive accuracy. In addition, the effect of operating conditions such as transmembrane pressure, pH, and temperature needs further exploration. The study also acknowledges fouling as a significant challenge in ultrafiltration, suggesting that integrating more sophisticated fouling models could enhance long-term predictive capabilities.

Species removal from aqueous radioactive waste by deep-bed filtration

The authors [5] presented two models for the separation of clay particles and radioactive suspended particles from aqueous suspensions: a deterministic filtration coefficient (FC) model and a stochastic two-state (TS) model. The deterministic model, based on mass balance equations, considers both attachment and detachment of particles, while the stochastic model employs a Markov-type connection between two states of particle evolution within the filtrating granular bed.

These models depend on macroscopic methods, which are easier to implement and require fewer assumptions compared to microscopic approaches. They enable the prediction of process performance under different operating conditions and provide a framework for optimizing deep bed sand filtration (DBSF) systems used in radioactive waste treatment. However, the models neglect the electrostatic forces, hydrodynamic effects, particle size distribution, and concentration profiles throughout the bed that may affect particle sedimentation. Although such models form a rigorous basis for the optimization of filtration processes, the scope of their application to the case of ultrafiltration can be improved by accounting for other factors, including electrostatic interaction, hydrodynamic force, and particle shape effects.

Further research should focus on refining the models to account for these complexities, ensuring greater accuracy and reliability in predicting filtration system performance.

Experimental and Modelling of Aqueous Radioactive Waste Treatment by UF

The study by [38] employs a 2^3 factorial design to investigate the effects of feed flow rate, operating pressure, and feed total suspended solids (TSS) on permeate flux and permeate TSS. By using regression equations, the study establishes a mathematical relationship between these process factors and response variables, demonstrating the application of statistical modeling in optimizing ultrafiltration processes.

A key attribute of this study is its use of factorial design, which enables the systematic examination of how multiple process factors interact. The model's predictive capabilities, demonstrated through regression equations (9) and (10) enable the estimation of permeate flux and total suspended solids (TSS) under various operating conditions, making it a useful tool for process optimization.

$$j_p = 118.25 + 4.50x_1 + 67.25x_2 - 8x_3 + 2.75x_1x_3 - 4.50x_1x_3 \tag{9}$$

$$TSS_p = 10.662 - 3.287x_1 - 2.688x_2 + 9.438x_3 - 2.413x_1x_3 - 2.413x_2x_3 \tag{10}$$

j_P is permeate flux, x_1, x_2, and x_3 are feed flow rate, operating pressure, and TSS respectively. TSS_p is permeate TSS.

However, the model has some weaknesses. It considers only three process factors, potentially oversimplifying the complex ultrafiltration process. The model does not account for membrane fouling, temperature variations, or pH effects, which are crucial in real-world operations. Furthermore, the study does not validate its model across different membrane types or long-term operation scenarios, limiting its applicability beyond the tested conditions.

By leveraging Equation (9), the study provides a quantitative tool for predicting permeate flux under different process conditions. This equation is essential for process optimization and design, allowing for precise adjustments to feed flow rate, operating pressure, and TSS levels to maximize ultrafiltration efficiency.

These models can be applied in industrial ultrafiltration processes to:

1. Optimize feed flow rate, operating pressure, and feed TSS to achieve desired permeate flux and quality.
2. Develop control strategies that adjust operating conditions in real-time to maintain optimal performance.
3. Predict system behavior under changing process conditions, ensuring consistent separation efficiency.

In order to enhance the applicability of these models, further research is needed with a focus on:

1. Examining process factors, such as membrane fouling, temperature effects, and pH disparities.
2. Validating the model across different membrane materials and ultrafiltration systems.
3. Assessing long-term membrane performance and repeated-use effects.
4. Developing hybrid models that incorporate machine learning techniques to improve predictive accuracy.

Modeling And Optimization of Removal of Strontium and Caesium from Aqueous Streams by Size Enhanced Ultrafiltration Using Chitosan Derivative

The study by [20] explores the removal of strontium Sr(II) and cesium Cs(I) from aqueous streams using size-enhanced ultrafiltration (SEUF) with carboxymethyl chitosan (CMCh) as a size-enhancing agent. By employing response surface methodology (RSM), the study optimizes process variables to maximize the percentage rejection and binding capacity of CMCh with Sr(II) and Cs(I). One major strength of the model is the use of RSM, which allows for the simultaneous optimization of multiple process variables while identifying key interactions between them. Additionally, the study demonstrates high removal efficiencies, achieving 99% rejection for both Sr (II) and Cs (I), making SEUF with CMCh a promising approach for metal ion removal.

Furthermore, the use of CMCh as a size-enhancing agent is a novel approach that offers an efficient method for radioactive wastewater treatment.

However, the model has certain limitations, including the fact that the study is conducted within a specific range of process variables, thereby limiting its applicability to other conditions. Despite its contributions, the study highlights several knowledge gaps, including the lack of investigation into the effect of additional process variables such as temperature and pressure on the removal efficiency of Sr(II) and Cs(I).

The study provides substantive knowledge into optimizing ultrafiltration processes for practical applications. SEUF with CMCh demonstrates the potential for removing radioactive contaminants from nuclear power plant effluents, while the RSM model can be leveraged to optimize process conditions to maximize rejection rates and binding capacity.

These models contribute to real-world ultrafiltration process optimization by enhancing decision-making through predictive models that simulate performance under different operating conditions. Additionally, they help maximize efficiency by adjusting key variables such as pH, polymer-to-metal (P/M) ratio, and concentration, while reducing operational costs by refining system parameters before full-scale implementation.

To enhance the applicability of the SEUF model, future research should investigate temperature and pressure effects to refine operational parameters, evaluate membrane reusability over multiple cycles to determine long-term efficiency, and conduct economic feasibility studies to assess cost-effectiveness for industrial-scale applications. While SEUF with CMCh shows high potential for radioactive wastewater treatment, further improvements could enhance its reliability by developing hybrid membranes that combine CMCh with nanomaterials to improve stability and adsorption efficiency. Additionally, incorporating machine learning models alongside RSM (hybrid model) could refine process predictions and real-time optimizations, while expanding the range of metal ions tested could explore broader applicability in wastewater treatment.

Application of Biosorbents in Hybrid Ultrafiltration/Sorption Processes To Remove Radionuclides from Low-Level Radioactive Waste

The paper by [25] presents a hybrid ultrafiltration/sorption process utilizing biosorbents such as alginic acid and sodium alginate for the removal of radionuclides from liquid radioactive waste. This approach offers several advantages, including high radionuclide removal efficiency of up to 97% for 85Sr and 72% for 137Cs, low energy consumption, and cost-effectiveness due to the use of inexpensive biosorbents. In addition, this method showcases the ability to remove radionuclides from complex media, thus making it a viable solution for radioactive wastewater treatment. However, the process also presents certain limitations. The system is highly sensitive to pH and ionic strength, affecting radionuclide retention coefficients, while the biosorbents may be prone to fouling, reducing permeate flux over time. Furthermore, the process may not be as effective in high-salinity environments, which limits its broad applicability.

Despite its effectiveness, the study identified several knowledge gaps that require further exploration. The impact of different types of biosorbents on radionuclide removal is not well understood, and the effects of fouling on permeate flux and radionuclide retention remain unclear. Additionally, the scalability of the hybrid ultrafiltration/sorption process for industrial applications is yet to be determined, necessitating further investigation into its long-term feasibility.

The hybrid ultrafiltration/sorption process can be optimized for real-world applications by refining operational parameters such as the biosorbent-to-metal ratio, pH, and ionic strength. Additionally, different biosorbents or modifications to existing ones could enhance adsorption efficiency, while fouling reduction techniques, such as using baffles or movable components in the filtration apparatus, could improve long-term performance.

Modeling uranium (II) removal from aqueous solution by micellar enhanced ultrafiltration using response surface methodology

[34] presents a study on the micellar-enhanced ultrafiltration (MEUF) process optimized using response surface methodology (RSM) to remove uranium from aqueous solutions.

The RSM model optimizes key process variables (pH, pressure, uranium concentration, SDS concentration) to achieve maximum uranium rejection and permeate flux, making it valuable for industrial applications. For example, the study identifies the best conditions for uranium removal (10 ppm uranium, 20.12 mM SDS, pH 6, 8 bar pressure). Furthermore, by modifying input variables, the model can be adapted for other heavy metal removals.

While RSM provides a strong predictive framework, future research could explore more advanced modeling techniques, such as artificial neural networks (ANNs) and support vector machines (SVMs), to improve accuracy in capturing complex interactions. Additionally, incorporating more process variables, such as membrane characteristics and operational stability factors, could enhance the robustness of the model. Future research could investigate the impact of additional variables, such as temperature, membrane type, and feed flow rate, on process performance.

Sorption-Assisted Ultrafiltration (SAUF) for Radionuclide Removal

[11] presented a study on the use of sorption ultrafiltration (SAUF) for the removal of radionuclides from contaminated water. This method utilizes clay-salt sludge (CSS), an inexpensive and readily available aluminosilicate sorbent, to enhance radionuclide removal. The effect of several technological parameters like pH, sorbent dosage, temperature, and feed rate on efficiency of removal was investigated. The efficiency of SAUF on the removal of cationic and anionic radionuclides has been shown; however, the SAUF method is less effective for anion elimination. As a solution to this, the authors propose that the incorporation of reducing agents, such as tin chloride or hydrazine, in the growth process may enhance the efficiency. Overall, the study concludes that SAUF using CSS is a promising approach for treating radioactive wastewater.

One of the major strengths of this method is its cost-effectiveness and environmental sustainability, as it repurposes industrial waste (CSS) as a sorbent. Furthermore, the process achieves nearly 100% removal efficiency for certain radionuclides, particularly when optimal conditions are met.

However, the method has certain limitations. Its efficiency is highly dependent on maintaining an optimal sorbent dosage, as deviations from 2.5 g/L can lower performance. Moreover, organic complexing agents present in contaminated water may stabilize radionuclides such as technetium-99m, preventing their effective removal. Additionally, membrane fouling may reduce flow rates over time, affecting overall performance.

The SAUF method offers a useful approach for radionuclide removal because it is cost-effective and has high efficiency. Further research is needed, though, to improve anion removal, address long-term operational challenges, and assess scalability for practical applications.

Ultrafiltration separation of Am(VI)-polyoxometalate from lanthanides

The study by [37] presents a novel ultrafiltration-based approach for selectively separating americium (Am) from lanthanides (Ln) using a polyoxometalate (POM) cluster with a vacant equatorial donor site. The POM cluster exhibits a high affinity for Am(VI) ions, allowing for their efficient separation from Ln(III) ions. This approach offers high separation efficiency, minimal secondary waste generation, and potential recyclability of the POM cluster, making it a promising alternative to conventional separation methods.

However, the synthesis of the POM cluster is complex, requiring precise reaction conditions. Additionally, its stability in aqueous solutions is limited, as it may degrade over time, affecting its long-term usability.

The study identifies several knowledge gaps, including the uncertainty of POM cluster stability under varying aqueous conditions (e.g., pH, temperature, and ionic strength); the need to assess the scalability of ultrafiltration for industrial actinide separation; the necessity of further exploration into the selective Am(VI) coordination mechanism to enhance cluster design and efficiency; and the lack of detailed research on the potential application of POM clusters for separating other actinides such as U(VI) and Np(VI).

This method holds potential real-world applications in nuclear waste management by efficiently removing americium from reprocessing waste streams, reducing environmental contamination as a more sustainable alternative to solvent extraction, and enabling the recycling of valuable actinides for reuse in nuclear fuel cycles.

To further develop this separation technology, future research should focus on scaling up the ultrafiltration process for large-scale nuclear waste treatment, exploring the stability of POM clusters under harsh conditions over long-time scale, performing the interaction mechanism involved in how Am(VI) is selectively coordinated to enable more efficient separation, the design of innovative composite POMs with improved stability and selectivity, including further experimental evidence such as kinetic analysis and spectral studies to fine-tune behavior prediction, development of sophisticate computational model for actinide-POM interaction understanding, and optimizing membrane design by testing different materials and filtration conditions to enhance retention rates and process.

A Hybrid Adsorption / Ultrafiltration Membrane Process for Removal of Cs-137 From Radioactive Wastewater Using Natural Clay Adsorbent

A novel approach to enhancing membrane-based separation is the Submerged Membrane Adsorption Hybrid System (SMAHS), which integrates adsorption and ultrafiltration to improve Cs-137 removal efficiency. [26] explored this technique, demonstrating its effectiveness in Cs-137 removal from radioactive wastewater. The process leverages the high adsorption capacity of natural Iraqi clay minerals, such as bentonite, which contains montmorillonite and palygorskite. These minerals exhibit strong adsorption properties, allowing for efficient capture of radioactive contaminants before membrane filtration.

The SMAHS model presents several advantages. It is environmentally friendly, cost-effective, and utilizes locally available materials. The process also facilitates the immobilization of radioactive waste by converting it from a liquid to a solid phase, simplifying long-term storage. The mathematical representation of the model follows a flux equation ($J = V/A \times t$) to determine membrane permeability and rejection efficiency. Under optimal conditions, the system achieved a 93.6% rejection rate for Cs-137 and a permeate flux of 105.6 $L/m^2.h$.

However, certain limitations exist, including the need for optimization of operational conditions such as stirring speed, transmembrane pressure, and clay dosage. Additionally, concerns about membrane fouling and the long-term stability of the clay minerals require further investigation. The potential scalability of this process for large-scale applications remains uncertain due to the necessity of continuous stirring and possible membrane clogging.

5. Discussion

Based on the reviewed modeling approaches and case studies (summarised in Table 5), several important insights can be drawn regarding their suitability and limitations in various UF scenarios.

It is clear that no single model is universally optimal. Each model performs best under specific operating conditions. For example, mechanistic models are reliable for high-solids, high-pressure environments but require detailed understanding of the system.

On the other hand, empirical/statistical models are easier to implement but may not generalize well outside their design space. Data-driven models adapt well to nonlinear and dynamic systems, particularly when temperature or other operational variables fluctuate.

Hybrid models combine physical insight with predictive flexibility, making them ideal for real-time control and complex system behavior. They are well suited for hazardous or data-limited or scarce environments where experimentation is limited.

Mechanistic models still dominate for system design despite their complexity. These models are essential when regulatory transparency and safety justification are required.

Statistical models thrive in optimization and not prediction. They are useful in optimizing operational variables like pH, transmembrane pressure, etc. However, they lack physical interpretability and extrapolation capability, thus making them suitable only for design of experiment (DOE) and single-point optimization, not real-time prediction.

Models applied in sludge-like or high-TSS environment (e.g., [26][12]) benefit from mechanistic or hybrid synergies due to fouling and rheological effects. Empirical models underperform in such scenarios due to oversimplified assumptions. Table 6 below shows the model selection table for UF of radioactive waste.

Mechanistic UF models, ML-based models in the field of UF prediction, and the representation of hybrid models in six performance parameters, i.e., accuracy, interpretability, real-time applicability, scalability, data processing intensity, and ease of implementation are shown below in Fig. 4. The assessment is qualitative (5 = highest, 1 = lowest), based on review studies. Hybrid models are ideal for scenarios that require accuracy, scalability, and real-time capability.

Table 5
Summary classification of UF models for Radioactive waste studied.

Model/Study	Model Type	Optimization/Tool	Primary Purpose	Key Focus Area
Mechanistic Models				
Hybrid Complexation-UF/NF [35]	Mechanistic	Parameter tuning via experiments	Process design & ion separation	Complexing agents & retention
Hanford UF Process [10]	Semi-empirical (Film model)	Experimental correlation	Capacity optimization	Solid's concentration vs throughput
Classical Filtration Models [13]	Mechanistic	Resistance-in-series; empirical fit	Feed condition optimization	Sodium molarity, solids
HLW Sludge UF [14]	Mechanistic	Empirical + model fit	Performance prediction	Rheology, pressure, flux
Complexation-UF [30]	Mechanistic (modified)	Validation via experimental data	Rejection prediction	Pore size distribution
POM-UF for Am (VI) [36]	Mechanistic	Spectroscopic validation	Selective separation	Actinide coordination
Empirical / Semi-Empirical Models				
Sizing UF Process [9]	Empirical (linear fit / Geeting's curve)	Process curve optimization	Unit design	Cake growth, surface area
SAUF with CSS [11]	Empirical	Experimental data	Radionuclide removal	Sorbent dose, pH, redox
SMAHS with Clay [26]	Empirical/Mechanistic	Experimental flux equations	Cs-137 rejection & flux	Adsorption + membrane hybrid
Statistical Models				
Experimental UF Optimization [38]	Statistical	Factorial design + regression	Process optimization	Total suspended solids (TSS) & permeate flux
SEUF with CMCh [20]	Statistical	RSM	Sr/Cs removal optimization	Binding capacity, pH, P/M ratio
MEUF for U Removal [34]	Statistical	RSM	Uranium rejection optimization	Sodium dodecyl sulfate (SDS) micelles, pressure, pH
Data-Driven & Hybrid Models				
ANN for VMD [6]	Data-driven (ANN)	BFGS Quasi-Newton	Mass transfer prediction	Feed & permeate conditions
Hybrid UF/Sorption with Biosorbents [25]	Hybrid empirical	Experimental design	Radionuclide removal	Biosorbent performance, pH

Note: UF - Ultrafiltration; NF - Nanofiltration; HLW - High-Level Waste; POM - Polyoxometalate; Am(VI) - Americium (hexavalent state); SAUF - Sorption-Assisted Ultrafiltration; CSS - Chitosan-Supported Sorbent; SMAHS - Sorption-Membrane-Assisted Hybrid System; Cs-137 - Cesium-137; TSS - Total Suspended Solids; SEUF - Surfactant-Enhanced Ultrafiltration; CMCh - Carboxymethyl Chitosan; RSM - Response Surface Methodology; Sr/Cs - Strontium/Cesium; P/M ratio - Polymer-to-Metal ion ratio; MEUF - Micellar-Enhanced Ultrafiltration; SDS - Sodium Dodecyl Sulfate; ANN - Artificial Neural Network; VMD - Vacuum Membrane Distillation; BFGS - Broyden-Fletcher-Goldfarb-Shanno (Quasi-Newton optimization algorithm).

Table 6
Model selection table for UF of radioactive waste.

Operational Condition / Goal	Recommended Model Type	Justification
High solids concentration	Mechanistic / FEM / Hybrid	Required for detailed fouling, rheology, and cake growth modeling
High transmembrane pressure (TMP)	Mechanistic / CFD / FEM	Physical models best capture compaction, pressure-driven flux loss
Variable temperature conditions	ANN / Hybrid (e.g., CFD + ANN)	Learns nonlinear effects of temperature on viscosity and transport
Low experimental accessibility	ANN / RSM / CFD-trained surrogates	Data-driven or surrogate models reduce need for hazardous lab work
Process optimization (pH, dose, TMP)	RSM / Regression / ANN	Effective in DOE, multivariable optimization without full physical modeling
Real-time monitoring/control	ANN / Hybrid ANN	Fast response, low computational load; useful in feedback control systems
Module or system design	Mechanistic / CFD / FEM	Best for geometry-sensitive, physically interpretable modeling
Prediction of fouling behavior	ANN / Hybrid (ANN + FEM/CFD)	Captures nonlinear time-dependent trends in fouling resistance
Selective ion separation modeling	Mechanistic / Equilibrium Isotherm	Useful for modeling complexation, sorption, and membrane rejection mechanisms
Integration with biosorbents / hybrids	Hybrid Empirical / Mechanistic	Combines sorption models with membrane transport

Table 7
Available dataset of reviewed study.

Study	Type	Public Access	Link/DOI
[6]	ANN code with dataset	yes	doi/full/10.1080/15569543.2020.1744659
[37]	Experimental dataset. XRD for Am (VI)-POM coordination.	Yes (available on request)	http://www.ccdc.cam.ac.uk/data_request/cif
[38]	Experimental dataset. TSS/flux optimization data for nuclear wastewater.	yes	DOI:10.37358/RC.18.5.6278
[26]	Experimental data	Requires author permission	https://doi.org/10.1016/j.cherd.2024.07.036

Table 8
Potential Research Directions.

Knowledge Gap	Description	Proposed Research Direction
Fouling prediction limitations	Existing models, especially classical ones, inadequately predict time-dependent fouling dynamics.	Integrate fouling kinetics into hybrid ANN-mechanistic models using real-time operational data.
Limited benchmark datasets	Most UF models rely on proprietary, small-scale experimental datasets, limiting reproducibility.	Develop and publish open-access, domain-specific UF datasets (e.g., Cs^+, Sr^{2+} systems).
ML model interpretability ("black-box" issue)	Data-driven models like ANN and LSTM lack physical transparency, making them difficult to validate.	Employ physics-informed neural networks (PINNs) and explainable AI (XAI) frameworks.
Scalability to industrial operations	Many models fail to extrapolate accurately under real plant variability (e.g., fluctuating pH, TMP).	Build hybrid digital twin systems trained on pilot-scale data with embedded physical constraints.
Radioactive-specific chemical considerations	Standard UF models neglect ion valency effects, radiolysis products, and extreme ionic strength.	Modify models to include radionuclide-specific behavior, redox reactions, and radiation damage.
Lack of real-time feedback and control mechanisms	Most models are static and do not support real-time optimization or anomaly detection.	Develop sensor-integrated, closed-loop UF control systems using adaptive ML models.
Underexplored membrane-material interactions	Interactions between membrane composition and actinides remain poorly characterized.	Conduct studies on hybrid membranes (e.g., clay, POMs) and model their degradation mechanisms.

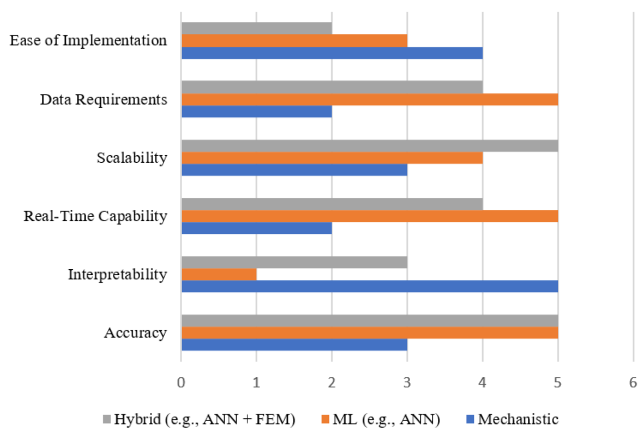

Fig. 4. Comparison of UF Model across Critical Dimensions.

The future of UF modeling for radioactive waste lies in hybrid integration, combining the physical robustness of mechanistic models with the speed, accuracy, and adaptability of data-driven approaches. In selecting a model, one must be guided by what the feed characteristics will be, the operational dynamics, and the modeling goals.

5.1. Data availability

The data supportive of the findings of this study are from previously published articles. While few studies offer open-source data, accessibility remains limited by proprietary and safety constraints. Table 7 below lists some of the studies with public datasets, along with the relevant access links or DOI.

5.2. Knowledge Gaps and Future Research Directions

Although there have been significant developments in mathematical modeling of ultrafiltration for radioactive waste treatment, there still exist a number of key areas where knowledge is lacking. Gaps exist in model formulation as well as data, operational, and system integration levels. Dealing with these problems is a prerequisite to achieving more comprehensive, predictive, and relevant UF modeling frameworks for industry. The categories of major gaps and potential research directions are summarized in Table 8.

To some extent these gaps will need to be addressed through interdisciplinary efforts between computational modeling, membrane engineering, and nuclear process design. By producing modular, benchmarked hybrid models with associated open datasets, the reliability, scalability and regulatory acceptance of UF in nuclear waste remediation will be markedly improved.

6. Conclusion

This review presents a comparatively outstanding and comprehensive overview of purely mechanistic, data-driven and model hybridization approaches in ultrafiltration of radioactive waste treatment. This research has also shown the potential of combining the classic (mechanistic) UF models with modern computational methods such as ANN and RSM to optimize the treatment of radioactive wastewater. Classic models are foundational, but hybrid methods greatly improve prediction and control. Not only do such models drive forward scientific knowledge, but they are also indispensable in efforts to create safer, more efficient, and more sustainable nuclear waste management systems.

Appendix A

TITLE-ABS-KEY (*mathematic* AND *model* AND *ultrafiltration*) AND (LIMIT-TO (SRCTYPE, "j")) AND
(EXCLUDE (SUBJAREA, "BIOC") OR EXCLUDE (SUBJAREA, "AGRI") OR EXCLUDE (SUBJAREA, "PHAR")
OR EXCLUDE (SUBJAREA, "IMMU") OR EXCLUDE (SUBJAREA, "BUSI") OR EXCLUDE (SUBJAREA, "SOCI")
OR EXCLUDE (SUBJAREA, "ARTS") OR EXCLUDE (SUBJAREA, "ECON") OR EXCLUDE (SUBJAREA, "DENT")
OR EXCLUDE (SUBJAREA, "PSYC") OR EXCLUDE (SUBJAREA, "VETE"))
AND (LIMIT-TO (LANGUAGE, "English")) AND (LIMIT-TO (DOCTYPE, "ar")).

Data availability

Data will be made available upon request.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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